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State of the Art

Evaluation of Subcritical Crack Extension under Constant Loading*

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Abstract

The determination of subcritical crack growth data is necessary for lifetime predictions of ceramic components. In this context the problem of measuring subcritical crack growth with natural cracks and macro-cracks is considered. It can be stated that the crack growth law for small natural cracks cannot be derived from measurements carried out with macro-cracks. For the determination of the v–K_I relation with specimens containing natural flaws different methods are applied. Especially, a lifetime method developed by the authors allows the determination of crack growth rates down to $1 \times 10^{-12}\,m\,s^{-1}$. Resulting v–K_I curves are reported for hot-pressed silicon nitride, Al_2O_3 and glass.

Die Bestimmung unterkritischer Rißwachstumsdaten ist für die Lebensdauervorhersage von keramischen Bauteilen notwendig. In dieser Arbeit wird das Problem der Messung des unterkritischen Rißwachstums mit natürlichen Rissen und makroskopischen Rissen untersucht. Man stellt dabei fest, daß das Rißwachstumsgesetz für kleine natürliche Risse nicht aus Messungen mit makroskopischen Rissen abgeleitet werden kann. Für die Bestimmung der v–K_I Beziehung an Proben, die natürliche Risse enthalten, werden andere Methoden angewendet. Vor allem erlaubt eine von den Autoren entwickelte Lebensdauermethode die Bestimmung von Rißwachstumsraten bis zu $1 \times 10^{-12}\,m\,s^{-1}$. Für heißgepreßtes Siliziumnitrid, Al_2O_3 und Glas werden gewonnene v–K_I Kurven angegeben.

L'estimation de la propagation sous-critique de fissures est nécessaire pour prévoir la durée de vie de pièces céramiques. On considère ici le problème de la mesure de la propagation sous-critique de fissures avec des fissures naturelles et des macro-fissures. On peut montrer que la loi de propagation des petites fissures naturelles ne peut être déduite des mesures réalisées sur des macro-fissures. On a appliqué différentes méthodes pour déterminer la relation v–K_I dans le cas d'échantillons contenant des défauts naturels. Les auteurs ont en particulier développé une méthode permettant de déterminer des vitesses de propagation allant jusqu'à $1 \times 10^{-12}\,m\,s^{-1}$. Les courbes v–K_I résultantes sont données pour un nitrure de silicium pressé à chaud, une Al_2O_3 et un verre.

1 Introduction

Different failure modes are responsible for failure and finite lifetimes of ceramic materials. The most important of them are:

1. spontaneous failure
2. subcritical crack growth under static load
3. cyclic fatigue
4. thermal fatigue
5. creep, creep crack growth, creep fracture

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Spontaneous failure occurs when the applied stress reaches the strength of the material or in terms of fracture mechanics when the stress intensity factor K_I of the most serious crack in a component reaches or exceeds the fracture toughness K_{Ic}. Therefore, the knowledge of K_{Ic} is necessary to assess spontaneous failure behaviour.

Delayed failure under moderate temperatures can be caused either by subcritical crack growth governed by the actual stress intensity factor K_I or by crack propagation under cyclic load governed by the stress intensity factor range ΔK and probably the R-ratio defined as the quotient of minimum and maximum K-value.

Thermal fatigue shows at least a combination of the failure modes mentioned before. Additional effects as for instance oxidation may also have an influence.

In the *high temperature* region, where a pronounced creep is present a structure can fail when excessive creep deformations become too large and the component stops functioning. In the creep range fracture can be caused by *creep crack growth* where the crack growth rates are governed by the C^*-integrals which are different for primary and secondary creep. *Creep fracture* describes the generation and accommodation of creep pores and their influence on fracture. In this paper subcritical crack growth under static loading is treated in detail.

2 Failure by Subcritical Crack Growth

2.1 General relations

The failure of ceramic components is often caused by subcritical crack propagation. In the range of linear-elastic fracture mechanics crack growth is governed only by the stress intensity factor K_I which describes the stresses near a crack tip

$$\frac{da}{dt} = v(K_I) \tag{1}$$

K_I is defined by

$$K_I = \sigma\sqrt{a}\,Y \tag{2}$$

where σ denotes the stress and a the depth of a crack in a structure, and Y is the geometric correction factor dependent on the shape of the crack and the component. If the crack growth behaviour can be described by a power law

$$v = AK_I^n \tag{3}$$

one obtains rather general *lifetime relation*

$$\int_0^{t_f} [\sigma(t)]^n \, dt = B\sigma_c^{n-2}\left[1 - \left(\frac{\sigma_f}{\sigma_c}\right)^{n-2}\right] \tag{4}$$

with

$$B = \frac{2}{AY^2(n-2)K_{Ic}^{n-2}} \tag{5}$$

If $(\sigma_f/\sigma_c)^{n-2} \ll 1$ a simplificated form results

$$\int_0^{t_f} [\sigma(t)]^n \, dt = B\sigma_c^{n-2} \tag{4a}$$

Equation (4) allows lifetime predictions based on pure subcritical crack growth for arbitrarily chosen time-dependent stresses.

2.2 Methods for determination of subcritical crack growth data

To allow correct lifetime predictions the relation (3) has to be known, especially for extremely low crack growth rates. Different methods of determining the v–K_I curves are available in the literature as:

—double-torsion (DT) method[1]
—double-cantilever beam (DCB) technique[2]
—controlled fracture test[3]
—dynamic bending strength test[4]
—lifetime measurements in static tests[5]
—modified lifetime method[6]

The three first procedures are carried out with macroscopic cracks on the order of several millimetres, but for lifetime predictions the crack growth behaviour of natural cracks of the order of 50 μm is of interest.

It is known that the lifetimes of components with small natural cracks cannot be predicted satisfyingly from v–K_I curves obtained with specimens containing a large crack. This holds especially for materials with a strong R-curve effect where the crack growth of large cracks is significantly influenced by increasing toughness, whereas the effect on crack growth behaviour of small natural cracks may be negligible.

It was earlier shown by Adams *et al.*[7] and recently by Chen *et al.*[8] that the crack growth law for natural cracks is in contrast to macro-crack results. It was found out that the exponents of the well-known power law are by a factor 4 and more lower for natural cracks compared with the exponents for macro-cracks of several millimetre size. This behaviour is in agreement with own results reported in Section 2.3.

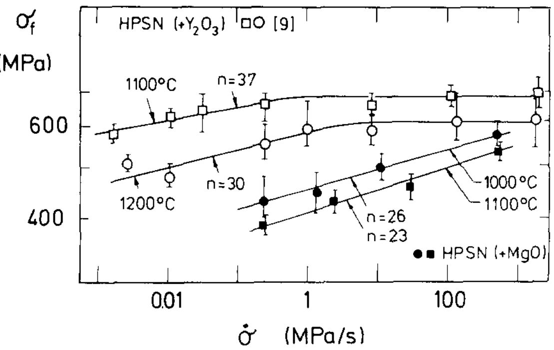

Fig. 1. Dynamic bending strength of hot-pressed silicon nitride.

2.2.1 The dynamic bending test

From measurements of bending strengths at different stress rates, $\dot{\sigma}$, one can evaluate n and B (or A). From eqn (4), one obtains

$$\sigma_f^{n+1} = B\sigma_c^{n-2}\dot{\sigma}(n+1)[1-(\sigma_f/\sigma_c)^{n-2}] \qquad (6)$$

For very high loading rates ($\dot{\sigma} \to \infty$) it results $\sigma_f \to \sigma_c$ For low loading rates it follows asymptotically

$$\sigma_f^{n+1} = B\sigma_c^{n-2}\dot{\sigma}(n+1) \qquad (7)$$

Before the dynamic bending strength results can be evaluated by eqn (7), it must be ensured that in the investigated range of $\dot{\sigma}$ eqn (7) is valid. Therefore, the evaluation of strength tests at only two stress rates is completely unsuitable.

Figure 1 shows results obtained by Keller[9] on Y_2O_3-doped hot-pressed silicon nitride (HPSN) at high temperatures. Both limit cases can easily be identified. A least-squares fit including all strength values would give absurd n-values. Additional disadvantages of this procedure are:

—The type of v–K_I relation has to be known.
—Inevitably, the bending strength is affected mainly by crack growth at a relatively high crack growth rate so that the crack growth parameters obtained are not necessarily characteristic of those crack growth rates which are of interest for lifetime predictions.

2.2.2 The lifetime methods

By combining eqns (1) and (2), the lifetime formula for the static load test ($\sigma = \text{const.}$) results as

$$t_f = \frac{2}{\sigma^2 Y^2}\int_{K_{Ii}}^{K_{Ic}} \frac{1}{v(K_I)} K_I \, dK_I \qquad (8)$$

Very often, the assumption of a power law is made to evaluate the integral in eqn (8). By introducing eqn (3) in eqn (8) and taking into consideration

$$K_{Ii}^{n-2} \ll K_{Ic}^{n-2}$$

the well-known *conventional lifetime relation*

$$t_f = B\sigma_c^{n-2}\sigma^{-n} \qquad (9)$$

results. As an application of this method static lifetime measurements from Ref. 10 are reported in Fig. 2 for hot-isostatically-pressed Al_2O_3 carried out in 4-point bending tests in a concentrated salt solution at 70°C. From the slope and the position of the least-squares straight line, the crack growth parameters were found to be

$$n = 20$$
$$B\sigma_c^{n-2} = 3{\cdot}25 \times 10^{45}\ \text{MPa}^{20}\text{h}$$
$$B = 0{\cdot}3914\ \text{MPa}^2\text{h}$$

Apart from the invalidity of eqn (9) for short lifetimes due to the neglection made during the derivation, the weakness of this method is always the special prescribed type of subcritical crack growth law.

The *modified lifetime procedure* is also based on eqn (8). Differentiation of eqn (8) with respect to the initial stress intensity factor K_{Ii} results in

$$v(K_{Ii}) = -\frac{2K_{Ic}^2}{Y^2\sigma_c^2 t_f}\frac{d(\ln K_{Ii}/K_{Ic})}{d(\ln t_f\sigma^2)} \qquad (10)$$

In the derivation, no special type of subcritical crack growth law is prescribed and no neglections relating to the upper limit of integration are made.

The needed change in K_{Ii} can be generated by introducing uniform small surface cracks, for instance by Knoop-indentation and varying the bending stress applied or by use of a fixed stress and making use of the scatter of the natural cracks. The first possibility is a very appropriate procedure if the initial size of the artificial cracks can be identified after the lifetime test.

The procedure of evaluation $v(K_I)$ is relatively simple. In a first series of tests, N samples are tested in dynamic bending tests at high stress rates in an inert environment to give the distribution of σ_c. The N-values of strength are arranged in an increasing order. In a second series, also involving N specimens, the lifetimes t_f were measured. The results are also arranged in increasing order. The νth value of lifetime $t_{f,\nu}$ is associated with the νth value of inert bending strength, $\sigma_{c,\nu}$. The latter is transformed into K_{Ii} using the relation

$$K_{Ii} = K_{Ic}\frac{\sigma}{\sigma_c} \qquad (11)$$

The lifetime data represented in Fig. 2 were re-evaluated and combined with inert strength data (for details see Ref. 10). The resulting crack growth rates are given in Fig. 3. The results for the single stress

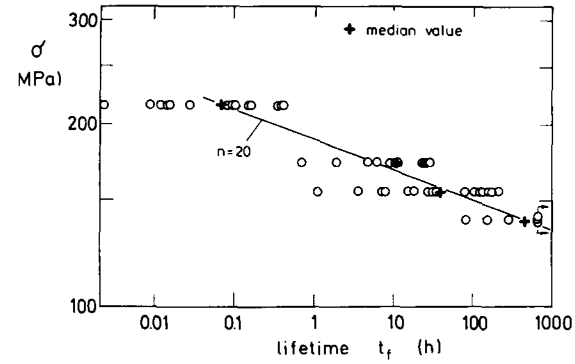

Fig. 2. Lifetimes of Al_2O_3 in a high concentrated salt solution.

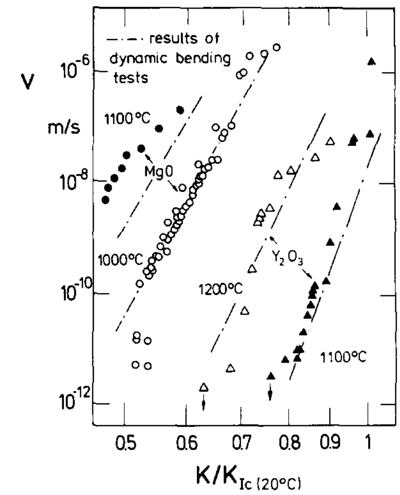

Fig. 4. Crack growth rates for HPSN at high temperatures.

levels are identical within the scatter. In this case, the data can be well described by a simple power law of type eqn (3). A least-squares fit yields an exponent of $n = 19$ in accordance to the conventional procedure mentioned before. In Fig. 4, high temperature results (circles and triangles) obtained for hot-pressed silicon nitride are compared with the results of dynamic bending tests[11] (dashed-dotted lines). There is excellent agreement between the two methods, both based on natural cracks.

2.3 Comparisons between small cracks and macroscopic cracks

The *static bending test with notched specimens* provides an appropriate way to determine the subcritical crack growth behaviour of macroscopic cracks. In a 3-point bending arrangement, the specimen is statically loaded with load P (less than necessary for spontaneous failure) and the displacement δ is measured in the centre of the supporting roller span S by a LVDT. If the material shows subcritical crack propagation, the displacement δ does not remain constant but will increase with time. The amount of additional displacement after the elastic response ($\Delta\delta$) can be recorded with a high resolution. Figure 5 shows a displacement versus time curve for an Al_2O_3 ceramic tested at 20°C in air.

Immediately after load application, a high displacement rate $\dot{\delta}$ appears which becomes reduced with increasing time, and only a short time before the specimen fails $\dot{\delta}$ arises again. The displacement increment $\Delta\delta$ is caused by a change of the compliance C which is a direct consequence of a crack extension Δa. It holds

$$\Delta C = P\,\Delta\delta \tag{12}$$

From the actual compliance, C, the crack depth, a, can be evaluated for any time. The only fracture mechanical quantity necessary for the evaluation of this static '*macroscopic crack growth test*' is the geometric function, Y, for the crack-load-configuration. The function Y for $S/W = 8$—based on numerical

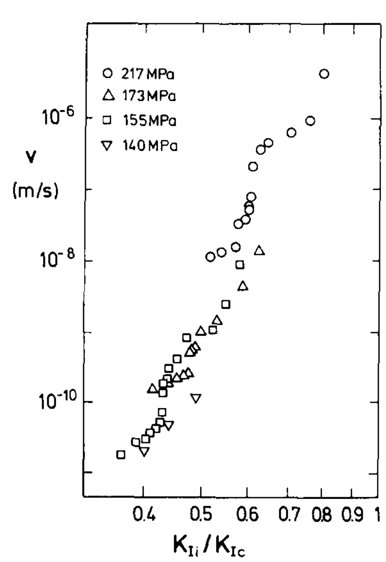

Fig. 3. Crack growth rates for Al_2O_3 in concentrated salt solution.

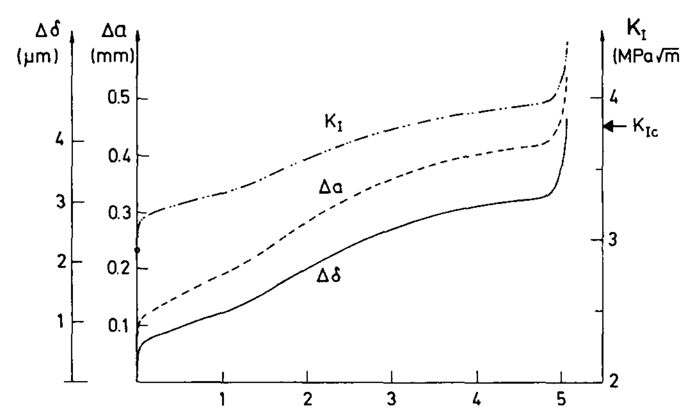

Fig. 5. Displacements, crack length and the actual stress intensity factor K_I.

results of Gross and Srawley[12]—can be expressed by

$$Y = \frac{1}{(1+2\alpha)(1-\alpha)^{3/2}} \times \left[1{\cdot}99 - \frac{\alpha(1-\alpha)}{(1+\alpha)^2}(1{\cdot}4925 + 0{\cdot}685\alpha - 2{\cdot}8325\alpha^2 + 2{\cdot}085\alpha^3 + 1{\cdot}35\alpha^4)\right] \quad (13)$$

The relation between relative crack depth, $\alpha = a/W$, and compliance, C, is simply given by

$$C = C_0 + \frac{9}{2}\frac{S^2}{W^2EH}\int_0^{\alpha} Y^2\alpha'\,\mathrm{d}\alpha' \quad (14)$$

where

$$C_0 = \frac{S^2}{W^2HE}\left[\frac{S}{4W} + \frac{(1+\nu)W}{2S}\right]$$

is the compliance of the unnotched bending bar; E is the Youngs modulus, ν denotes the Poisson ratio, and H is the width of the specimen. A numerical evaluation of the integral in eqn (14) yields after curve fitting

$$C = C_0 + 1{\cdot}99^2\frac{9}{4}\frac{S^2}{W^2EH}\frac{\alpha^2}{(1-\alpha)^2(1+3\alpha)} [1 - 0{\cdot}8953\alpha + 0{\cdot}69655\alpha^2 - 0{\cdot}38523\alpha^3] \quad (15)$$

within $\pm 0{\cdot}2\%$ for $0 \leq \alpha \leq 0{\cdot}95$.

Knowledge of crack depth, a, allows computation of the actual stress intensity factor by eqn (2). In Fig. 5 additionally, the time-dependent crack depth and the stress intensity factor are plotted. A comparison of the K_I-values with the fracture toughness of 3·8 MPa$\sqrt{m}$ (obtained in a fast load rate controlled 3-point bending test) illustrates a significant R-curve

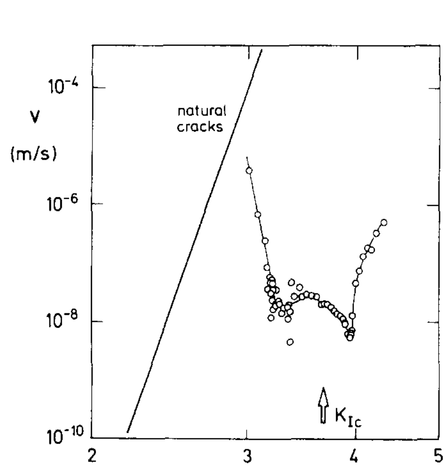

Fig. 6. v–K_I curves for natural cracks compared with the results of notched bending bars.

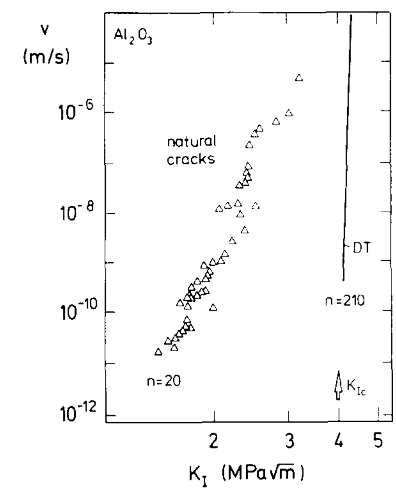

Fig. 7. v–K_I curve for natural cracks compared with double-torsion results.

behaviour. Finally, the v–K_I curve results by taking the time derivative of the $a(t)$-curve. In Fig. 6, this macro-crack v–K_I curve is plotted in addition to the v–K_I relation obtained from static lifetime measurements on specimens with natural crack population.

It must be stated that only at the beginning of the macro-crack bending test do the high crack rates—expected from the natural cracks—occur. After a small amount of crack extension, the crack growth rate drops for several magnitudes. From this result, it becomes evident that lifetime predictions for specimens with natural cracks cannot be based on 'macro-crack results'.

A second example may support this statement. From the results of Figs 2 and 3, an unusually low crack growth exponent, $n \simeq 20$, was concluded (Ref. 10. The identical material was also investigated in DT-tests by Hermansson.[13] His resulting v–K_I curves showed very high n-values in the range of $150 \leq n \leq 410$ in complete contrast to the natural crack results. In Fig. 7, the mean-value curve reported in Ref. 13 is compared with the data of Figs 2 and 3. An obvious indication on the existence of a strong R-curve effect is the difference of the K_{Ic} value which was 4·0MPa$\sqrt{m}$ for the edge-notched bending specimen and 6·0–6·4 MPa$\sqrt{m}$ for the DT-test.

3 Conclusions

In the first part of this paper, different methods of the determination of the relation between crack growth rate and stress intensity factor K_I are reviewed. In metallic materials, the v–K-relation for stress corrosion cracking or the $\mathrm{d}a/\mathrm{d}N$–ΔK-relation for cyclic loading are always determined with simple plate specimens with straight through-the-thickness cracks. The transformation of these results to real cracks is performed by calculating the appropriate stress intensity factors for these cracks. This transformation is possible under the asumption that both the local stress- and strain-distribution at the crack tip in laboratory specimens and in the cracked component are governed by the stress intensity factors.

In ceramic materials, the situation should be similar. There are, however, two effects which may contribute to different behaviour between the macro-cracks in the specimens and the natural microflaws in the component. First, the natural flaws such as pores or cracked or debonded inclusions are not necessarily cracks with sharp tips as required for fracture mechanics application. Such cracks may grow from these flaws during the first part of the lifetime. In the DT- or DCB-specimens, the extension of already-existing cracks is measured.

The second effect is related to the R-curve behaviour under increasing load tests. As was shown by Knehans and Steinbrech,[16] the interaction of the crack borders behind the crack tip reduces the effective stress intensity factor. This effect increases with crack extension. Therefore, the applied stress intensity increases with crack extension. A similar effect obviously occurs during subcritical crack extension.

The effective stress intensity factor, K_{eff}, can be written as the difference between K_{appl} from the applied stress and K_{int} from the interaction between the crack surfaces:

$$K_{\mathrm{eff}} = K_{\mathrm{appl}} - K_{\mathrm{int}}$$

Whereas K_{appl} increases with the square root of the total crack length, K_{int} increases with the crack extension Δa. Therefore during crack extension from a saw cut, a decrease in K_{eff} can first be expected. This leads to a decreasing of the crack growth rate with increasing K_{appl}, as shown in Fig. 6. For micro-cracks, a similar behaviour may occur. The amount of crack extension during most of the lifetime, however, is very small.

From these considerations, a simple relation between crack growth rate and applied stress intensity factor cannot be expected. Nevertheless, such a relation is useful and necessary for lifetime predictions. If this relation is determined under similar loading situations and with the same natural flaws as in a real component, then the overall description of the complicated effects of initiation of a crack from a natural flaw and the crack border interaction with a unique v–K-curve may be possible. It is therefore strongly recommended to use specimens with natural flaws for the determination of the v–K-curve. The modified lifetime method as described in this paper is especially recommended because it is able to measure—indirectly—very low crack growth rates.

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